MEM6810 Engineering Systems Modeling and Simulation
工程系统建模与仿真

Theory Analysis

Lecture 6: Verification and Validation of Simulation Models

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Introduction

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- One of the most important and difficult tasks facing a model developer is the *verification* and *validation* of the simulation model.
- **Verification** (检验): building the model correctly.
  - Is the desired model implemented correctly in computer (e.g., simulation software, programming language)?
- **Validation** (验证): building the correct model.
  - Is the simulation model an accurate representation of the real system?

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- Repeated steps in model building:
  - Observe the real system.
    – collecting data
    – consult domain experts
  - Construct a conceptual model.
    – assumptions about the system
    – hypotheses about the parameter values
  - Implement an operational model.
    – using simulation software or programming language

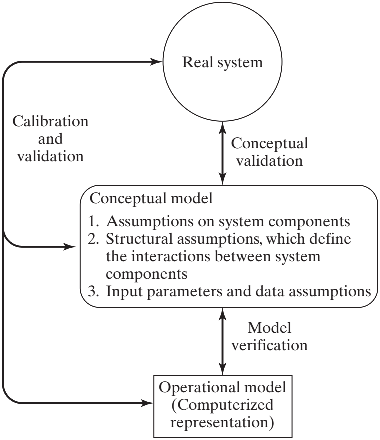

Figure: Model Building Process (*from Banks et al. (2010)*)

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  - To increase the model's credibility to an acceptable level.
- It is the job of the model developer to work closely with the end users throughout the period of development.

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    – In a trace, the **state** of the simulated system is displayed just after each event occurs and is compared with hand calculation.
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  - If the operational model is animated, verify that what is seen in the animation imitates the actual system.

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  - If the utilization of one station is unreasonably low (or high), it may indicate some possible mistakes in parameters and/or model logic.
  - It may be possible to set the input and/or logic (e.g., Poisson arrival, exponential service time, FCFS) so that the complex queueing network becomes a Jackson queueing network.
    - Compare the analytically solved limiting distribution and L, W, L_Q, W_Q with the simulation results.

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  - Use some sets of data for calibration and other independent sets for "final" validation.

Validation of Simulation Models

- No model is ever a perfect representation of the actual system.
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  - The level of model detail should be consistent with the type of data available.

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- Widely used approaches in the validation process:
  - Build a model that has high face validity (表面效度).
  - Validate model assumptions.
  - Compare the model input-output transformations with the actual system's data.

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- The potential users of a model should be involved:
  - to ensure that a high degree of realism is built into the model;
  - to check a model's face validity e.g., whether the model behaves in the expected way when one or more input variables is changed?

    – Example: In most queueing systems, if the arrival rate of customers increases, it would be expected that server utilization, queue length, and waiting time will increase (although by how much might be unknown).

- General classes of model assumptions:
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  - Data assumptions: interarrival times of customers (peak or slack periods), service times (commercial or personal accounts).
    - Verify data reliability with bank managers.
    - Test homogeneity (when combining data) and correlation.
    - Test the goodness of fit for hypothesized distribution.

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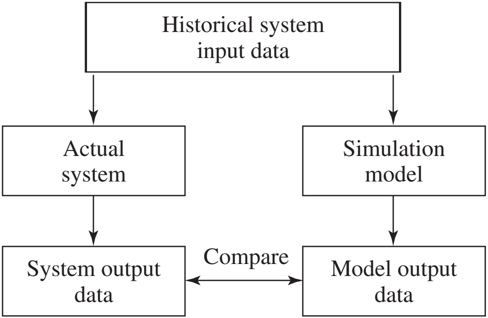
- Use the main responses of interest as the primary criteria for validating a model.
  - If the model is used later for a different purpose, it should be revalidated in terms of the new responses of interest.

- Input-output validation using historical input data.



- Example: Simulation of a store with 2 parallel servers and 2 waiting lines.
  - It opens at 9 AM and closes at 5 PM every working day.†
  - The primary interested response (i.e., measure) is the average waiting time of customers during peak hour (11 AM to 1 PM).
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  - Simulation model is built based on some assumption and simplification.
    - In actual system, customers may switch waiting lines spontaneously.
    - In simulation model, we assume that customers choose the shortest line when they come and will not change later.
  - Before using the simulation model to answer our question, we first need to validate the model.

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- For each day $i = 1, \ldots, K$, we can compute every customer's arrival time and service time (A_{ij}, S_{ij}), and his waiting time W_{ij}, $j = 1, 2, \ldots$. We can also get the average waiting time during peak hour, W_i.

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- Suppose the data are reliable, and we can model the arrival (perhaps by a nonhomogeneous Poisson process) and service time with satisfying goodness of fit.
- The remaining issue is on the validity of the structural assumption, i.e., the queue behavior.

- We conduct input-output validation using historical input data.
  - Run the simulation model for K replications, and for each replication $i = 1, \ldots, K$, historical data $\{(A_{ij}, S_{ij}) : \ j = 1, 2, \ldots\}$ were fed into the model as input.
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  - The simulated average waiting time during peak hour is Y_i, $i = 1, \ldots, K$.
  - Compare system output data $\{W_1, \ldots, W_K\}$ with model output data $\{Y_1, \ldots, Y_K\}$.
  - We are interested in knowing if the two sets of output data are equal statistically.

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Input Data Set	System Output W_i	Model Output Y_i	Observed Difference d_i	Squared Deviation from Mean $(d_i - \bar{d})^2$
1	W_1	Y_1	$d_1 = Y_1 - W_1$	$(d_1 - \bar{d})^2$
2	W_2	Y_2	$d_2 = Y_2 - W_2$	$(d_2 - \bar{d})^2$
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.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
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  - If the K input data sets are fairly homogeneous, then $d_1, \ldots, d_K$ can be considered as iid realizations of D.
  - It is reasonable to assume that D follows normal distribution.
    - Each Y and W is an average over customers.
    - By the Central Limit Theorem, Y, W, and D are approximately normally distributed when the customer number is large.

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  - The p-value is $\mathbb{P}(|T| \geq |t|) = 2\,\mathbb{P}(T \geq |t|)$.
  - Under significance level α, reject H_0 if p-value $< \alpha$, or $|t| > t_{K-1,\,1-\alpha/2}$, where $t_{K-1,\,1-\alpha/2}$ is the $(1-\alpha/2)$-quantile of t distribution with $K-1$ degrees of freedom.

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- One may also consider to construct a confidence interval for μ to test H_0.
- When only one data set (instead of K) is available,† one needs to use some time-series approaches to compare model output data with system output data.

† A common situation in nonterminating simulation (see more details in Lec 7).

- A Turing test can be used in addition to statistical test, or when no statistical test is readily applicable.

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  - If the manager cannot distinguish between fake and real reports with consistency, conclude that the test gives no evidence of model inadequacy.